LIQUID EVAPORATION FROM CAPILLARIES OF VARIABLE SECTION

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UDC 532.72

An analytical description is presented of the process of liquid evaporation from capillaries of variable cross section at temperatures close to the boiling point.

In real capillary-porous bodies the pore form is usually not cylindrical, and drying processes cause this form to change with time due to structural changes in the skeleton of the body. The present study will investigate evaporation of a liquid from capillaries of variable section, the radius of which $r = r(x, l(t))$. The gas mixture flow in the capillary will be assumed one-dimensional and steady state. The binary gas mixture into which evaporation occurs consists of vapor molecules (the first component) and molecules of the gas forming the surrounding medium. The gas molecules do not undergo a phase transition on the liquid surface. The process will be assumed isothermal, and the Knudsen number significantly less than unity.

The molecular flux densities for vapor G_1 and gas G_2 in a capillary of circular section will be described in the following manner:

$$G_1 = nc_1 \frac{r^2(x,\ l)\,kT}{8\eta}\,\frac{dn}{\partial x} + nD\,\frac{dc_1}{dx}\,, \tag{1}$$

$$G_2 = nc_2\,\frac{r^2(x,\ l)\,kT}{8\eta}\,\frac{dn}{dx} + nD\,\frac{dc_2}{dx}\,. \tag{2}$$

In the general case the capillary radius r is a function of coordinate x and position of the phase boundary l. For constant r, over the entire course of the process Eqs. (1), (2) transform to the corresponding ones for a cylindrical channel [1]. We choose for the positive direction of the coordinate along which vapor–gas mixture flow takes place the direction from the capillary mouth to the meniscus. The boundary conditions for Eqs. (1), (2) have the form

$$c_1(x = 0) = c_{10},\ c_1(x = l) = c_{1l}. \tag{3}$$

where $c_{1l} = p_{1s}/p$.

Values of the relative concentrations of the external gas at the capillary mouth (c_{20}) and meniscus (c_{21}) are found from the condition

$$c_1 + c_2 = 1. \tag{4}$$

As was shown in [2], viscous flow in a channel of radius r may be considered as steady state given the condition $x > 0.1\,\mathrm{Re}\cdot r$. Then at a distance $|dx|$ (since $\mathrm{Re} = vr/\nu$) the inequality

$$|dx| \gg 0.2\,\mathrm{Re}\,|dr|$$

must be fulfilled, whence

$$\left|\frac{dr}{dx}\right| \ll \frac{5}{\max\{\mathrm{Re}\}}\,. \tag{5}$$

Equation (5) is a necessary condition for definition of the region of applicability of Eqs. (1), (2).

For steady-state flow the flow rate of vapor and gas through the channel section per unit time is a constant quantity: $\Gamma_1 = G_1S(x,\ l) = \mathrm{const}_1$, $\Gamma_2 = G_2S(x,\ l) = \mathrm{const}_2$. Then the total flow of vapor gas mixture $\Gamma = \Gamma_1 + \Gamma_2$ can be written with consideration of Eqs. (1), (2) in the form

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Kalinin Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 4, pp. 647-651, October, 1984. Original article submitted June 21, 1983.

0022-0841/84/4704-1222$08.50

$$\Gamma = \pi n \, \frac{r^4(x, l)\, kT}{8\eta} \, \frac{dn}{dx} \, . \tag{6}$$

In deriving Eq. (6) it was considered that $S(x, l) = \pi r^2(x, l)$. Integration of Eq. (6) from p_0 to p_1 with the replacement $n = p/kT$ leads to the expression

$$\Gamma = \frac{p_l^2 - p_0^2}{16kTU_4(l)} \, , \tag{7}$$

where

$$U_4 = U_\alpha\,(\alpha = 4); \; U_\alpha = \frac{1}{\pi} \int\limits_0^l \frac{dx}{r^\alpha(x, l)\eta} \, .$$

In the integration it was assumed that the viscosity of the mixture is equal to some average value (for example averaging could be performed for partial pressures of vapor and gas above the meniscus and at the capillary mouth [3]), which is constant for a fixed phase boundary position.

Equation (6) also permits us to transform Eqs. (1), (2):

$$\Gamma_1 = c_1\Gamma + n\pi r^2(x, l)\, D \, \frac{dc_1}{dx} \, , \tag{8}$$

$$\Gamma_2 = c_2\Gamma + n\pi r^2(x, l)\, D \, \frac{dc_2}{dx} \, . \tag{9}$$

The product nD is a constant [1, 4]. Integration of Eqs. (8), (9) leads to the expressions

$$\Gamma_1 = \frac{c_{1l} - c_{10} \exp\left[-\dfrac{\Gamma U_2(l)}{nD}\right]}{1 - \exp\left[-\dfrac{\Gamma U_2(l)}{nD}\right]} \, \Gamma, \tag{10}$$

$$\Gamma_2 = \frac{c_{2l} - c_{20} \exp\left[-\dfrac{\Gamma U_2(l)}{nD}\right]}{1 - \exp\left[-\dfrac{\Gamma U_2(l)}{nD}\right]} \, \Gamma. \tag{11}$$

The condition that the gas does not pass through the phase boundary allows us to express the total flow using Eqs. (11) and (4) as

$$\Gamma = \frac{nD}{U_2(l)} \ln\left(\frac{1 - p_{10}/p_0}{1 - p_{1s}/p_l}\right) . \tag{12}$$

Setting Eqs. (7) and (12) equal to each other, we obtain a transcendental equation for the unknown pressure value above the meniscus p_l:

$$\frac{p_l^2 - p_0^2}{16kT\eta U_4(l)} = \frac{nD}{U_2(l)} \ln\left(\frac{1 - p_{10}/p_0}{1 - p_{1s}/p_l}\right) . \tag{13}$$

As follows from Eq. (13), the value of the total pressure above the phase boundary in a variable section channel depends not only on the channel radius and the partial pressures of the binary mixture components (as is true of a cylindrical capillary [5]), but is also a function of the meniscus coordinate.

Substituting Eq. (12) into Eq. (10) gives us the expression

$$\Gamma_1 = \frac{nD}{U_2(l)} \ln\left(\frac{1 - p_{10}/p_0}{1 - p_{1s}/p_l}\right) . \tag{14}$$

Here $p_l \neq p_{1s}$. For sufficiently large radii $(U_2(l)/U_4(l) \to \infty)$, it follows from Eq. (13) that $p_l \to p_s$. In this case, as is evident from Eq. (7),

$$\Gamma_1 = \frac{p_{1s}^2 - p_0^2}{16kT\eta U_4(l)} \, . \tag{15}$$

The expressions for the flow, Eq. (14) or (15), together with Eq. (13), allow determination of the law of change of meniscus position with time t in a capillary of length L. This dependence is found from the law of conservation of liquid mass

$$-\frac{dM}{dt} = \Gamma m_1, \qquad (16)$$

where $M = \pi\rho Q$; $Q \equiv \int_0^{L-l} r^2(y, l)\,dy$. Hence

$$t = \frac{\pi\rho}{m_1}\int_l^0 \frac{\varphi(l)\,dl,}{\Gamma}, \qquad (17)$$

where $\varphi(l) \equiv dQ/dl$. Equation (17) allows calculation of the time required for liquid evaporation from a channel of variable section.

We will consider some special cases of change in capillary dimensions. Thus, if the radius changes identically over the entire length during the evaporation process, then

$$\varphi(l) = 2r(l)(L-l)\frac{dr}{dl} - r^2(l),$$

$$U_4(l) = \frac{l}{\pi r^4(l)}, \quad U_2(l) = \frac{l}{\pi r^2(l)}.$$

Equation (13) for p_l transforms to the corresponding equation presented in [1, 6] for each fixed value.

For a conical capillary which expands as the phase boundary retreats into the medium, described by the equation $r(x) = r_0 + b(l)x$ (where r_0 is the initial cylindrical capillary radius, $b(l)$ is the conicity, which varies during the evaporation process), the functions $\varphi(l)$, $U_4(l)$, $U_2(l)$ are equal to:

$$\varphi(l) = -[r_0^2 + b(L-l)]^2 + (L-l)^2\frac{db}{dl}\left[r_0 + \frac{2}{3}b(L-l)\right],$$

$$U_4(l) = -\frac{1}{3\pi b}\left[\frac{1}{(r_0+bl)^3} - \frac{1}{r_0^3}\right],$$

$$U_2(l) = \frac{l}{\pi r_0(r_0+bl)}.$$

In particular, for liquid evaporation from a conical capillary with constant conicity over its entire length (for r_0 we may take the value of the radius at the capillary mouth) in the latter equations we take $b = \text{const} > 0$ and $db/dl = 0$.

As an example, we will consider a conical capillary with constant low conicity ($bL \ll r_0$). We also assume that the radius in all sections is sufficiently great that $p_l \to p_{1s}$. In this case the evaporation time is given by

$$t = \frac{8\rho kTl^2\eta}{m_1 r_0^2(p_{1s}^2 - p_0^2)}\left(1 - \frac{2bL}{r_0} - \frac{2bl}{3r_0}\right).$$

The relative time for evaporation from a conical capillary as compared to the evaporation time t_0 from a cylindrical capillary of radius r_0 is given by the expression

$$\frac{t}{t_0} = 1 - \frac{2bL}{r_0} - \frac{2bl}{3r_0}.$$

Thus, for $r_0 = 10^{-5}$ m, $L = 10^{-2}$ m, $b = 10^{-4}$ the total evaporation time for a conical capillary comprises 74% of the evaporation time from a cylindrical capillary of radius r_0.

NOTATION

G_i, molecular flow density of component i in the gaseous phase; $i = 1$, vapor; $i = 2$, surrounding gas; $c_i = p_i/p$; p_i, partial pressure of component in mixture; $p = p_1 + p_2$; $n = n_1 + n_2$; n_i, molecular concentration of component in mixture; μ, capillary radius; x, coordinate; k, Boltzmann's constant; T, temperature; η, dynamic viscosity coefficient of mixture; D, binary diffusion coefficient; p_{1s}, saturated vapor pressure at process temperature; $Re = vr/\nu$, Reynolds number; v, hydrodynamic velocity of gas mixture; ν, kinematic viscosity coefficient; Γ_i, molecular flux of component i; $S(x, l) = \pi r^2(x, l)$, channel cross-sectional area; t, time; L, capillary length; M, liquid mass; m_1, molecular mass of vapor substance; ρ, liquid density; subscripts: 0, l, values at capillary mouth and meniscus, respectively.

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NONSTEADY HEAT AND MOISTURE TRANSFER IN CAPILLARY-POROUS COLLOIDAL BODIES WITH CONVECTIVE DRYING

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UDC 66.047.37

A mathematical model describing the distribution of moisture content in the region of a moist state of capillary-porous colloidal bodies is proposed.

Formulation of the Problem

The drying of moist capillary-porous colloidal bodies is a typical nonsteady process occurring in the presence of transfer-potential gradients. The moisture-transfer potential for the given bodies is assumed to be the chemical potential of water vapor as a function of the temperature and partial pressure of the vapor. In the hygroscopic region, it may be expressed using the temperature and moisture content of the body. Taking this into account, the system of differential equations

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t + \frac{\varepsilon r_0}{C}\frac{\partial U}{\partial \tau}, \tag{1}$$

$$\frac{\partial U}{\partial \tau} = D\nabla^2 U + D\delta\nabla^2 t, \tag{2}$$

describing the interrelated phenomena of heat and moisture transfer, was derived in [1]. Numerous solutions of this system of equations with different boundary conditions are found to be in good agreement with the experimental results of [2-7].

For wet bodies, the chemical potential is equal to the potential of free water, i.e., it is constant and cannot be used as the moisture-transfer potential. This explains the considerable deviation in the moisture-content distribution obtained from the solution of Eq. (2) from the experimental values in [2-7].

A mathematical model derived under the following assumptions is proposed to describe the moisture distribution in capillary porous bodies:

1) the transfer of capillary-bound water is not diffusional;

2) capillary-bound water is characterized by a density ρ, which is equal to the mass of this water per unit volume of the body;

3) nonsteady transfer of the moisture occurs under the influence of the combined action of the motive forces (pressure and temperature gradients, capillary potential, etc.). It is assumed that the resulting flux may be expressed using the rate of transfer $\vec{v}_{cap}$ and the density ρ by the equation

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Higher Wood-Technology Institute, Sofia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 4, pp. 651-657, October, 1984. Original article submitted June 9, 1983.

0022-0841/84/4704-1225$08.50